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A STUDY OF UNSYMMETRICAL-LOADING CONDITIONS

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SUMMARY

The force variation along the wing span under combined normal and angular accelerations is considered. Nondimensional curves of section load, shear, and moment are given for:

- (1) The air load that produces a normal acceleration.
- (2) The accompanying wing weight and normal inertia loads.
- (3) Aileron and gust air loads that produce angular acceleration.
- (4) The angular inertia load of the wing.

The required aerodynamic load distributions have been obtained by use of wing theory and the wing inertia distributions are based on an analysis of wing-weight data.

Several examples are included to illustrate the effect of wing taper and aileron span on the total shears and moments at any section along the span.

INTRODUCTION

Although the design of most strength members of an airplane structure is determined by symmetrical-loading conditions in which the accelerating forces and couples lie in the plane of symmetry, the design of some few members is determined by unsymmetrical-loading conditions in which the accelerating forces and couples lie outside the plane of symmetry and, in addition to lateral accelerations, angular accelerations may occur about either the fore-and-aft or the vertical axis.

One of the more important of the unsymmetrical-loading conditions is considered to occur when an airplane is subjected to a combined vertical acceleration and an angular acceleration about the X (fore-and-aft) axis. This condition may prove to be the critical one for (1) the members of the wing-cabane structure, (2) engine nacelles, (3) the members near the wing-fuselage attachment points, and (4) the members located near and supporting large weight items attached to the wing.

Existing rules relating to the design of the wings in the unsymmetrical-loading condition are concerned only with specifying the amount of the unbalanced air-load moment acting on the wings, no particular attention being paid to whether the airplane can reach the angular acceleration implied. In addition, the angular inertia of the wings is usually neglected except for the inertia effects of the engines and other concentrated items housed within the wing, which may be taken into account. In both instances, the requirements appear to be conservative. A more rational method of design would be to specify the maximum angular acceleration that a given airplane may attain in flight either through use of the controls or from encountering gusts and to allow for the wing inertia.

The aim of the present paper is to consider briefly the unsymmetrical-loading conditions, treating the airplane as a rigid body symmetrical about a vertical plane and taking into account as far as possible the effect of wing taper and aileron span on the shear and the moment at each section along the span. There being many variables, the discussion has been limited to the presentation in nondimensional form of the important air and inertia load distributions and to a brief discussion of various trends that may occur with changes in wing weight and weight distribution. The necessary air-load distributions have been obtained from theoretical computations, and the wing weight and the inertia distributions have been obtained from an analysis of wing-weight-distribution data. The only air-load distributions considered in this paper are those for initially untwisted wings and those due to deflecting ailerons equally and oppositely.

SIGN CONVENTIONS AND ELEMENTARY CONCEPTS

The origin of coordinates will be, unless otherwise stated, located at the center of gravity of the airplane.

The positive directions of the X, Y, and Z axes will be, respectively, backward, to the left, and upward; forces acting on the airplane in these directions will be considered positive. These conventions are illustrated in figure 1. Positive moments tend to rotate the positive X axis into the positive Y axis, etc., in cyclic order.

When the resultant of all the distributed forces (of any particular load distribution) outboard of a given spanwise station is positive, the shear at that section is positive. Thus, in the beam direction, positive bending moments cause compression in the upper fibers of the wing when it is considered as a beam.

In figure 2(a) is shown a wing-fuselage combination that is subjected simultaneously to a normal acceleration a in the Z direction and an angular acceleration α about the X axis. Any element of mass dm located at the distance r from the origin will have two components of linear acceleration, one equal to $a_{z.c.g.}$, the acceleration of the origin, and the other equal to $r\alpha$. The part $r\alpha$ has the two components

$$a_{z\alpha} = y\alpha \quad (1)$$

$$a_{y\alpha} = -z\alpha \quad (2)$$

perpendicular and parallel to the wing span, respectively. Thus, the total linear acceleration of any element normal to the wing span varies directly with the horizontal distance of the mass element from the origin; and the linear acceleration parallel to the span is, unless dihedral is present, independent of the spanwise position. Similar relations exist when the airplane is subjected to simultaneous linear acceleration in the X direction and an angular acceleration about the Z axis.

In addition to the foregoing accelerations, each element is subjected to a centripetal acceleration directed toward the origin or the center of gravity and equal to $\omega^2 r$; where ω is the angular velocity. Inasmuch as the angular velocities encountered in roll and yaw are small, being less than 2 radians per second, and also because this maximum angular velocity is unlikely to occur at the instant that the largest combined values of normal and angular acceleration exist, the effect of centripetal acceleration is neglected.

In a one-wheel landing (fig. 2(b)), the airplane actually rotates about the point of contact with the ground. The rotation may, nevertheless, be assumed to take place about the center of gravity of the airplane in accordance with the principle that: The motion of a rigid body in a plane may be considered at any instant as a combination of rotation about any axis O , in the plane of motion of the body, and a translation of the body that gives to each particle the same velocity and acceleration which the axis O would have at that instant. Thus, it is possible to treat the case of the airplane landing on one wheel in the same manner as an airplane subjected to a normal acceleration in the Z direction and an angular acceleration about the X axis. (See fig. 2(b).)

The normal component of the linear acceleration of the center of gravity in landing is then

$$a_{z\text{c.g.}} = a_{z\text{wheel}} - y_t \alpha \quad (3)$$

and the horizontal component is

$$a_{y\text{c.g.}} = a_{y\text{wheel}} + h\alpha \quad (4)$$

Again, neglecting centripetal components, each particle dm of the wing is subjected to the acceleration components

$$a_z = a_{z\text{wheel}} + (y - y_t)\alpha \quad (5)$$

$$a_y = a_{y\text{wheel}} - (z - h)\alpha \quad (6)$$

perpendicular and parallel to the wing span, respectively.

Instead of dealing with each particle dm , it is possible to take an entire wing section extending inward from the wing tip to a general section J and to consider the forces acting on it. Equations (1) to (6) still apply; but a distance \bar{y} , from origin to center of gravity of the outboard portion, is then used instead of y .

NORMAL ACCELERATION COMBINED WITH

ANGULAR ACCELERATION IN ROLL

Effects of ailerons.— If it is required to determine the effect of a combined normal and angular acceleration

only on the total design loads of the wing fittings and adjacent fuselage members, the following must be known: First, the maximum aerodynamic forces and moments that may act on the airplane; and, second, the separate weights and moments of inertia of the complete airplane and of the fuselage. The forces and the moments that must reach the fuselage fittings to give these accelerations are then readily determined. If the load and the moment variation along the span for various loading conditions is required, however, the procedure becomes a little more involved.

Let J (fig. 2(a)) be a general section at which it is desired to obtain the total shear and moment during combined angular and normal accelerations. In flight, the portion of the wing outboard of the section J has various component load distributions that act on it in the beam direction. These distributions for the case of aileron operation, are:

1. An air-load distribution symmetrical about the plane of symmetry, which produces the normal acceleration or load factor. This distribution is termed the "A" distribution. Although this distribution may, in general, be composed of several components, this paper considers only the component associated with an untwisted wing.
2. An air-load distribution due to operating the ailerons, which gives to the airplane an angular acceleration when the ailerons are first deflected.
3. A wing-weight plus wing-inertia distribution, which is symmetrical about the plane of symmetry and which results from the normal acceleration.
4. An angular-inertia distribution, which results from the angular acceleration.

In addition to the distributed loads, there may be concentrated loads acting at various positions along the span. The effect of the loads on the total shear Q and the total moment M at a section may be separately considered, however, as may the effect of any one of the component load distributions.

The first of the mentioned load distributions is given in figure 3(a) for wings of various taper ratio. The type A load distributions have been obtained from the theoretical results that are presented in table I of reference 1. For structural estimates, they may be considered to apply to untwisted wings having rounded tips and aspect ratios from 8 to 12, approximately. The load

per foot run l_{a_z} at any station $y/\frac{b}{2}$, due to an air load that produces a load factor n in the Z direction, is obtained by multiplying the ordinates of figure 3(a) by nW/b , where b is the airplane span and W is the airplane design weight.

The air-load distribution due to deflecting, equally and oppositely, ailerons that cover various amounts of the span ($b_a/b = 1.0, 0.75, 0.5, \text{ and } 0.25$, where b_a is the aileron span) is shown in figure 4(a). These distributions are for a tapered wing ($\lambda = 2:1$) with rounded tips having an aspect ratio A of 10, although they may be used with good accuracy for aspect ratios of 8 to 12. They have been derived from theoretical aileron load-distribution curves given in figure 2 of reference 2. The load per foot run l_{a_α} at any station $y/\frac{b}{2}$, due to deflecting the ailerons an amount sufficient to produce an angular acceleration α , is obtained by multiplying the ordinates of figure 4(a) by $I_x\alpha/b^2$, where I_x is the airplane moment of inertia about the X axis (lb sec² ft).

The results given by equations (3) and (4) of reference 3 can be extended to express the normal wing weight and the inertia distribution by the nondimensional form given in figure 5(a). These distributions principally apply to the wing structural weight and include that part of the wing weight distribution due to ribs, spars, wing covering, wiring, control tubes, ailerons, flaps, and other distributed loads but do not include large concentrated loads that may be attached to the wing. From figure 5(a) the effective dead-weight load of the wing per foot run l_{w_z} at any station corresponding to a normal load factor n is obtained by multiplying the ordinates of the appropriate curve of figure 5(a) by fnW/b , where f is the ratio of wing structural weight to the total weight. Usually the factor f varies between 0.1 and

and 0.2, depending upon the type of construction and the design load factor.

The fourth distribution, that is, the angular inertia distribution, which is given in figure 6(a), has been derived from the results given in figure 5(a). The angular inertia load per foot run $l_{w\alpha}$ at any station due to an angular acceleration α is obtained by multiplying the ordinates of figure 6(a) by the factor $\frac{fW}{g}\alpha$, where g is the acceleration of gravity.

The nondimensional shear distributions corresponding to the air-load and the wing-inertia distributions have been obtained by an integration of the load curves and are given in figures 3(b) to 6(b). The ordinates of the shear curves are sometimes quite small near the tips; they are tabulated in tables I to IV.

The shear at any station due to any one of the component distributions, that is, Q_{a_z} , Q_{a_α} , Q_{w_z} , and Q_{w_α} , may be found by multiplying the appropriate ordinates of the curves of figures 3(b) to 6(b) (or the values given in the tables) by the various factors that appear in the denominators.

The corresponding moment distributions are shown in figures 3(c) to 6(c) and have been obtained by an integration of the nondimensional shear curves. The ordinates for the moment curves are also given in tables V to VIII.

When concentrated loads are assumed to act, the resultant shear and moment distributions in the beam direction are modified by the addition of shear and moment "blocks" and moment "triangles." The shear block will extend from the position $y_c/\frac{b}{2}$ of the concentrated load into the wing center or root with a magnitude equal to $-W_c(n + \frac{\alpha}{g} y_c)$, where W_c is the weight of the concentrated load in pounds. The moment triangle is zero at $y_c/\frac{b}{2}$ and has a maximum value at the wing center equal to $-W_c y_c(n + \frac{\alpha}{g} y_c)$.

In addition to the additive moment triangle there is a moment block, which extends inward from the load to the wing center, of magnitude $-I_0\alpha$, where I_0 is the moment of inertia of the weight item about an axis through its own center of gravity and parallel to the X axis of the airplane. For concentrated items that are some distance out on the wing, this moment is small with respect to the moment caused by the linear part of the acceleration and may often be neglected.

Of the quantities involved in the computation for the shear and the moment at any section, the values of I_X and α are the most difficult to determine in a given case. The value of the angular acceleration that a given airplane may attain by an instantaneous deflection of the ailerons depends upon a number of variables, such as the wing plan form, the aileron span, the aileron chord, the aileron deflection, and the type of aileron flap used. The last three variables may be conveniently represented by Δc_l , the increase in the section lift coefficient with a given flap deflection.

Now

$$L = I_X\alpha = C_l q S b \quad (7)$$

where

- L applied aileron rolling moment.
- C_l aileron rolling-moment coefficient.
- q dynamic pressure.
- S wing area.

But C_l can also be given by

$$C_l = \frac{2}{Sb} \int_0^{\frac{b}{2}} c_{l_b} c y dy \quad (8)$$

where

- c wing section chord.
- c_{l_b} section lift coefficient due to aileron distribution.

Now let $c_{l_b} = L_b \Delta c_l \frac{S}{bc}$ (as in reference 1) and also let $y = k \frac{b}{2}$ so that $dy = \frac{b}{2} dk$. Then

$$\alpha = \frac{qb^3 \Delta c_l}{2I_{xA}} \int_0^1 L_b k dk \quad (9)$$

where A is wing aspect ratio b^2/S . Values of

$\int_0^1 L_b k dk$ have been derived from the theoretical

aileron distributions given in reference 2 for tapered wings having ailerons of constant flap-chord ratio covering various portions of the span; the variation is given in figure 7.

The theoretical values do not approach the experimental ones owing to the facts that (a) the ailerons cannot be instantaneously deflected, (b) there is a lag in lift with deflection, and (c) the wing is flexible. Flight experiments of a Navy fighter monoplane (the XF13-C) indicate that, even with the most rapid rate of aileron deflection possible, only one-half the angular acceleration predicted by equation (9) is obtained. With much larger airplanes, it seems that a further slight reduction might be in order because of greater inertia of the controls and higher forces required. The necessary values of the section characteristic Δc_l for a given type of aileron flap may be obtained from wind-tunnel tests. Such data are given in a number of N.A.C.A. reports, of which reference 4 is an example.

The moment of inertia I_X of the airplane about the X axis may roughly be considered to be made up of four parts:

1. The moment of the wing structure.
2. The moment of the power-plant installation.
3. The moment of the fuselage and its contents.

4. The moment of other large concentrated items such as landing gear, gas tanks, and bombs housed in the wing.

Thus

$$I_x = I_{\text{wings}} + I_{\text{eng}} + I_{\text{fus}} + \sum \frac{W_c}{g} y_c^2 \quad (10)$$

Several average values have been derived to evaluate the various items of equation (10). An analysis of existing two-engine and four-engine airplanes indicated that the average engine positions are about $0.24 \frac{b}{2}$ and $0.32 \frac{b}{2}$ for the inner and the outer engines, respectively. Further, on the assumption that the mass of the engine, the propeller, and the accessories is $f_{\text{eng}} \times \frac{W}{g}$, where f_{eng} is the ratio of the weight of the engine-propeller-accessories combination to the total airplane weight, then

$$I_{\text{eng}} \approx 0 \quad (\text{one-engine airplanes})$$

$$I_{\text{eng}} \approx 0.0144 f_{\text{eng}} \frac{W}{g} b^2 \quad (\text{two- and three-engine airplanes})$$

$$I_{\text{eng}} \approx 0.020 f_{\text{eng}} \frac{W}{g} b^2 \quad (\text{four- and five-engine airplanes})$$

The value of the quantity f_{eng} usually lies between 0.125 and 0.200.

Reasonable values of the moment of inertia of the fuselage I_{fus} may be determined by assuming that the mass of the fuselage is disposed as a cylinder of uniform density with a maximum diameter equal to its largest diameter. If the mass of the fuselage and its contents is

denoted by $f_{\text{fus}} \frac{W}{g}$ and if it is further assumed that the maximum diameter of the fuselage is, on the average, one-twelfth of the wing span, then

$$I_{\text{fus}} \approx 0.00087 f_{\text{fus}} \frac{W}{g} b^2$$

The value of the quantity f_{fus} usually varies between

0.55 and 0.75, depending upon the placing of such parts of the useful load as gas, oil, and armament.

The weight of the wing structure being assumed equal to fW , the moment of inertia of the wings about the X axis may be written

$$I_{\text{wings}} = F \frac{fW}{g} b^2$$

Values of F , derived from an extension of the results given in reference 3, are as follows:

Taper ratio	F
1:1	0.0800
4:3	.0685
2:1	.0573
4:1	.0468
∞	.0362

Collecting terms, the moment of inertia of the airplane about the X axis may be expressed by

$$I_X = \frac{W}{g} b^2 \left(Ff + \begin{Bmatrix} 0 \\ 0.0144 \\ 0.020 \end{Bmatrix} f_{\text{eng}} + 0.00087 f_{\text{fus}} \right) + \Sigma \frac{W_c}{g} y_c^2 \quad (11)$$

where the $\Sigma \frac{W_c}{g} y_c^2$ term accounts for the moment of

inertia of any concentrated loads such as gasoline, oil, and bombs that may be housed in the wing. In the use of

of equation (11), the quantity $f + f_{\text{eng}} + f_{\text{fus}} + \Sigma \frac{W_c}{W}$

should equal 1.0 after values have been assigned to the various factors. Assigning average values to these factors reveals that the inertia of the wings is roughly 2 to 3 times that contributed by outboard engines and is 12 to 20 times that of the conventional fuselage.

Although the design values of the normal load factor n have been pretty well established for the various airplane classes, little is known concerning the actual

magnitudes of the angular accelerations that are attained and still less is known of the simultaneous values of angular and normal acceleration that may occur. In any case, the maximum angular acceleration for a particular airplane is entirely dependent upon how fast and how far the control system allows the pilot to deflect the ailerons. There being little chance that the maximum angular and normal accelerations will occur simultaneously even on highly maneuverable airplanes, a maximum value of the load factor n would not be used with a maximum value of the angular acceleration. In later examples, conservative but reasonable values of n and α will be assumed.

If the quantities n , α , I_X , b_a/b , b , λ , f , and W are known, the total shear at any section $y/\frac{b}{2}$, due to a load factor n and an angular acceleration α (caused by the aileron), can be found from the equation

$$Q_z = \left(\frac{Q_{az}}{nW} \right) nW + \left(\frac{Q_{a\alpha}}{I_X \alpha / b} \right) \frac{I_X \alpha}{b} - \left(\frac{Q_{wz}}{fnW} \right) fnW - \left(\frac{Q_{w\alpha}}{fW/b\alpha} \right) \frac{fW}{g} b\alpha \quad (12)$$

where the various terms in the parentheses can be obtained from figures 3(b) to 6(b) or from table I to IV.

Similarly, the bending moment in the beam direction at any spanwise station is given by

$$M_z = \left(\frac{M_{az}}{nWb} \right) nWb + \left(\frac{M_{a\alpha}}{I_X \alpha} \right) I_X \alpha - \left(\frac{M_{wz}}{fnWb} \right) fnWb - \left(\frac{M_{w\alpha}}{fW/b^2\alpha} \right) \frac{fW}{g} b^2 \alpha \quad (13)$$

The values of the various terms in parentheses may be either read from figures 3(c) to 6(c) or taken from tables V to VIII.

Effect of gusts.— Although gusts of all configurations and sizes may be encountered in flight, it is usual for design purposes to consider gusts having a linear gradient and reaching a maximum value U_{max} in a horizontal distance H . When this type of gust simultaneously envelops both wings, it produces a normal load factor n the value of which is given by an equation of the form

$$n = 1 + \frac{K m \rho U_{max} V}{2 W/S} \quad (14)$$

where

- U_{max} an assigned maximum gust velocity.
- m slope of airplane lift curve, radian measure.
- ρ mass density of air.
- K a gust reduction factor to take care of gust gradient, lag in lift, and wing loading.

If instead of striking both wings the gust should strike only one wing, an angular as well as a normal acceleration would result. This type of gust may be considered equivalent to a uniform one of magnitude $U_{max}/2$ plus a uniform unsymmetrical gust of magnitude $\pm U_{max}/2$. (See fig. 8.) The normal load factor accompanying this type of gust would be one-half that given by equation (14) and the angular acceleration would be equal to that produced by deflecting full-span ailerons equally and oppositely an amount sufficient to produce an angle-of-attack change on each wing equivalent to $\pm U_{max}/2V$. The value to be used, instead of Δc_l in equation (9), is $5.6 \frac{U_{max}}{2}/V$, where the quantity 5.6 represents an average value of the section lift-curve slope.

For the foregoing gust conditions, the load, the shear, and the moment distributions are given in figures 3 to 6 and tables I to VIII. Thus, in order to obtain the shears and the moments, equations (12) and (13) are applicable without modification.

Another elementary shape of gust is the trapezoidal one. This type may be considered to be composed of the two components shown in figure 8. The first part produces the normal load factor n whose value is given by equation (14) and the second part produces an angular acceleration of magnitude

$$\alpha = \frac{qb^3}{2 I_x A} \frac{U_{max} - U_{av}}{V} F' \quad (15)$$

Values of the factor F' have been derived from results given in reference 2 and are as follows ($A = 8$ to 12 , approximately):

Taper ratio	F'
1:1	1.100
4:3	.825
2:1	.583
4:1	.380

The load per foot run l_{ag} , the shear Q_{ag} , and the moment M_{ag} at a point due to the trapezoidal gust are given in figure 9 as ratios with quantities which are assumed to be known or which can be found.

The total shear and moment at a point may be found by substituting Q_{ag} and M_{ag} for $Q_{a\alpha}$ and $M_{a\alpha}$ in equations (12) and (13). Values of $\frac{Q_{ag}}{I_X\alpha/b}$ and $\frac{M_{ag}}{I_X\alpha}$ are also tabulated in tables IX and X, respectively.

One-wheel landing.- In the case where the airplane lands on one wheel as shown in figure 10, it is assumed that, at the instant of landing, the angular velocity is zero and the wing has an air load $f_a W$ acting on it. This air load is assumed to be symmetrical about the plane of symmetry. The factor f_a represents the part of the airplane weight taken up by the air load on the wings at the instant of landing and is usually in the neighborhood of 0.75. In addition to the air load, there are assumed to be forces $Y = n_{1y}W$ and $Z = n_{1z}W$ applied at the wheel. Under these forces a combined angular and normal acceleration results. The distributed loads, shears, and moments shown in figures 3, 5, and 6 hold with only slight modification to the ordinates.

The quantity nW is replaced by $f_a W$ in figure 3; in figure 5 the value of n (normal wing weight plus inertia) to be used is obtained from the equation

$$n = n_{1z} - \frac{\alpha}{g} \gamma t \quad (16)$$

The value of $\frac{\alpha}{g}$ to be used with figure 6 and in equation (16) is obtained from the following equation:

$$\frac{\alpha}{g} = \frac{n_{1z}y_t - n_{1y}h}{k_X^2 + h^2 + y_t^2} \quad (17)$$

where k_X is the radius of gyration of the airplane about the X axis.

The effect of any concentrated loads along the span on the shear and the moment distributions may be taken into account as before, if the values of n and α/g given by equations (16) and (17) are used.

If the airplane lands on two wheels in such a manner that the left wheel has the force $n_{z_l}W$ acting on it and the right wheel has the force $n_{z_r}W$ acting on it, a combined normal and angular acceleration will also result. Equations (16) and (17) then become

$$n_{c.g.} = n_{z_l} + n_{z_r} - \frac{\alpha}{g} y_t \quad (16')$$

Neglecting the side forces or assuming, in this case, that they are zero,

$$\frac{\alpha}{g} = \frac{(n_{z_l} - n_{z_r}) y_t}{k_X^2 + h^2 + y_t^2} \quad (17')$$

APPLICATION OF RESULTS TO A NUMBER OF

HYPOTHETICAL AIRPLANES

The previous charts, together with the equations given, enable a fairly general estimate to be made of the total beamwise shears and moments at any station due to both distributed and concentrated loads. In order to show the effect of the various component load distributions on the shears and the moments, a number of examples have been worked out for some typical airplanes. The first examples are for a series of hypothetical cantilever low-wing monoplanes (A_1 to A_5) whose only differences are in aileron span, moments of inertia, and wing taper. The airplane

weights are the same but the moments of inertia, owing to the differences in wing taper, are slightly different. The fuselages, however, have the same moment of inertia in each case. The characteristics for these airplanes are given in table XI, together with the values of the load factor and the angular acceleration used. The angular accelerations given have been computed from equation (9) using a value of Δc_l of 0.4, corresponding roughly to the section increment in lift coefficient that would be obtained with 0.2-chord ailerons deflected about 8° . Owing to the different aileron spans, moments of inertia, and wing taper, the angular accelerations α of airplanes A_1 to A_5 due to a given aileron deflection are computed to be as follows:

	A_1	A_2	A_3	A_4	A_5
α	4.00	3.69	3.28	1.75	5.54

The component load, shear, and moment distributions, as well as the net distributions, are given in figures 11, 12, and 13 for these airplanes. The distributions on the left-hand side of these figures are proportional to the load factor n and those on the right-hand side are proportional to the angular acceleration α . Thus, an idea of the effect of combinations of n and α other than those used in table XI may be obtained. The net distributions shown are for the particular combinations of n and α that are given in the table.

It can be seen from figure 11 (right) that the air load due to the ailerons and the angular inertia loading are, in addition to being of opposite sense, roughly of the same order of magnitude so that in this case only a very small part of the initial unsymmetrical forces is carried into the fuselage. Further, it may be seen that nowhere along the span does the sum of the aileron and the angular inertia load appreciably exceed 40 pounds per foot run, which is less than 5 percent of the corresponding net symmetrical load; consequently, the effective change in span loading caused by the addition of the unsymmetrical loads (fig. 11, right) to the net symmetrical loads would produce only slight irregularities in the final effective load distribution. In any case, these slight irregularities in the final loading would be smoothed out in the corresponding shear and moment curves. Thus, the magnitude of the additive unsymmetrical shear and moment is of little importance in this conventional type of airplane for the

range of wing tapers and aileron spans that may be used. A comparison of the quantities in figures 4 and 9 shows that this statement also applies to the total shear and moments in the beam direction arising from the elementary gust shapes that were considered. Since the moments of inertia of the fuselages are the same in these examples, the extra angular accelerating moments applied at the fuselage fittings are proportional to the angular accelerations previously listed. These extra moments may, however, be split up differently for the various airplanes into shear forces and bending moments, as is indicated in figures 12 and 13. These differences are, however, of little importance because the original differences caused by the unsymmetrical loads are themselves small.

The effect of varying the wing structural weight factor f within normal limits (0.05 to 0.25) on the shear and the moment along the span is relatively unimportant for this type of airplane because most of the inertia is in the wings and an increase in f produces, for a given aileron deflection, a nearly proportional decrease in the angular acceleration. In the limiting case of a cantilever airplane where the weight is distributed along the wing so that the symmetrical air load and the wing weight would cancel at each section (that is, $f = 1.0$), the exact unsymmetrical loading becomes important. For this limiting case it would be possible, theoretically, to vary the chord of the ailerons along the span so that the aileron air-load distribution would be the same as the angular inertia distribution. This distribution would tend to make the gust and the landing conditions the critical ones.

The foregoing analysis for the cantilever wing without concentrated loads indicates that no really large unsymmetrical loads are likely to exist on conventional wings due either to operating the ailerons or to encountering the elementary gust shapes considered. Thus it seems that a reasonable procedure, as far as the beamwise load at any station is concerned, might be to neglect the effects of the unsymmetrical loading on the total shear and moment along the span, making an allowance only for the additional torsional moments introduced by deflecting the ailerons. In the design of the wing-fuselage fittings and the fuselage transverse cross bracing for flight conditions, the only extra additive moments required are those necessary to give to the fuselage the angular acceleration caused by a given aileron deflection or gust. The

fuselage moment of inertia for this case can be conservatively computed by assuming the fuselage to be a homogeneous circular cylinder with a diameter equal to the maximum fuselage width.

If the angular acceleration and the vertical distance from the center of gravity to the plane of the wing are sufficiently large, the additional force tending to slide the wing through the fuselage should also be taken into account. In a flight condition, this force is equal to $-f \frac{W}{g} z\alpha$ (see equation (2)) but, since the airplane may conceivably be sharply banked at the time of maximum angular acceleration, a more conservative value would be

$$\text{Sliding force} = -f \frac{W}{g} (1 + z\alpha) \quad (18)$$

The sliding force due to angular acceleration in a one-wheel landing is equal to $-f \frac{W}{g} (h + z)\alpha$ where $\frac{\alpha}{g}$ may be obtained from equation (17).

An illustration of how concentrated loads affect shears and moments under combined normal and angular acceleration, diagrams of load, shear, and moment are given in figure 14 for a hypothetical two-engine airplane (B) with the landing gear at the engines. The pertinent geometric characteristics and the various quantities assumed are listed in table XI. It can be seen (fig. 14) that, for airplane B, the sum of the unsymmetrical components (aileron load and angular inertia) is a slightly larger percentage of symmetrical components than was the case for airplane A. Outboard of the engines (for example 0.6 b/2), the net shear and moment are increased (or decreased) as much as 7 percent owing to the addition of the unsymmetrical components. This percentage, which is somewhat larger than that in the previous series of airplanes, is partly due to the choice of the values of n and α and partly to the fact that a relatively larger amount of the angular inertia is located near the center. This location requires that a proportionately larger amount of the accelerating moment reach these sections.

The effect of the concentrated loads is to cause the shear inboard of the load on the downward accelerated

wing to change in such a manner as to oppose the angular acceleration applied by the ailerons. The necessary moment to be applied at the fuselage to produce the angular acceleration in the direction of the aileron deflection is obtained from the change in bending moment, as is shown in the lower right of figure 14. Decreasing the aileron span and deflecting the ailerons so as to obtain the same acceleration would change the shear-moment relations at the fuselage, as was indicated in figures 12 and 13 for airplanes A_2 , A_4 , and A_5 . Keeping the angular acceleration the same, other things being equal, requires a smaller aileron deflection with an increase in span. Increasing the aileron span, however, spreads the aileron load more uniformly over each half of the wing, causing the moment arm to move inward so that, for the same rolling moment, larger shear forces will be transmitted to the wing root.

Although the combination of normal and angular accelerations has a relatively small effect on the shears and the moments inboard of a concentrated load, the effect may be large on the members that carry the concentrated load. In the case considered, it amounts to an increment of load factor of $1570/1900$ or 0.828 .

Airplanes A_2 and B were chosen to illustrate the effect of a one-wheel landing on the loads, the shears, and the moments, and the following assumptions were made: (1) The airplane is level and horizontal; (2) there is no aileron displacement; (3) there is no initial rotation; and (4), at the instant of landing, the wings are supporting three-quarters of the airplane weight. The necessary dimensions and the values of the load factors used are listed in table XI where it will be noted that the side-wise load factor is negative (acts to right).

Using equation (17), the values of the angular acceleration are computed to be

$$A_2: \alpha = 32.2 \frac{3 \times 6.5 - (-0.6)(-6.0)}{(6.09)^2 + (-6)^2 + (6.5)^2} = 4.44 \text{ radians/second}^2$$

$$B: \alpha = 32.2 \frac{1.75 \times 9 - (-0.35)(-8.0)}{(8.17)^2 + (-8)^2 + (9)^2} = 1.97 \text{ radians/second}^2$$

From equation (16), the load factors at the center of gravity are

$$A_2: n = 3.0 - \frac{4.44 \times 6.5}{32.2} = 2.104$$

$$B: n = 1.75 - \frac{1.97 \times 9}{32.2} = 1.20$$

The component distributions of the load, the shear, and the moment corresponding to the foregoing values of n and α are illustrated in figure 15. It will be noted that, with the combinations of the quantities chosen for airplane A_2 , the shear and the moment inboard of the landing gear are practically unaffected by the distributed loads and are almost entirely due to the normal and the side forces at the wheel. In fact, if the torsion due to inertia and air load were neglected, the design of the wing members inboard of the gear, for the one-wheel landing, could be proceeded with by assuming that all the angular and the linear inertia arose from the fuselage and that all distributed loads could be omitted.

The preceding statement holds, however, only in the case considered. It may be seen for airplane B that, with different values of n , α , and f , the effects of the distributed loads do not always cancel. The contribution of the distributed loads, however, to the shear and the moment inboard of the wheel is unlikely to exceed appreciably ± 10 percent of that arising from the wheel load even when the extremes, that is, fairly high air loads and light wings or zero air load and heavy wings, are employed.

Although the examples given have dealt entirely with the cantilever monoplane, the nondimensional curves given for the distributed loads could be directly used for the case of the braced monoplane. This statement also applies to all of the equations, except for equations (12) and (13) for the shear and the moment where it would be necessary to introduce appropriate terms for the strut reactions. The normal values of the strut load can be easily found whether the wing is continuous across the span or is hinged at the fuselage. The curves and the equations could also be used for biplanes provided that additional allowances are made for the relative efficiency of the wings, difference in weight between the wings, and possibly different aileron spans on each wing.

YAWING ACCELERATIONS

Although the combination of normal acceleration and angular acceleration in roll is of primary interest in this paper, the factors and the curves given may also be used to estimate the loads in the wing when there is angular acceleration about the vertical axis. Angular accelerations, that is, yawing moments, may occur in flight as a result of either rolling or yawing velocities, aileron deflection, sideslipping, or from an abrupt movement of the rudder. On the ground, yawing moments may occur as a result of unequal forces acting on the wheels during landing or taxiing.

As the design rules now stand, practically all of the chordwise bracing in the wing is likely to be determined by either the limiting speed steady-flight condition or the high angle-of-attack load-factor condition and the only parts likely to be designed by a yawing-acceleration condition are the bracing near the wing-fuselage attachments or the cabane structure. For this purpose, it is essential to know what conditions produce the largest angular accelerations as well as the relative inertias of the wing and the fuselage with their contents.

From a consideration of the various flight conditions in which angular accelerations in yaw could occur, those conditions due to rolling velocity and yawing velocity are likely to be less severe than those due to an abrupt rudder kick or side gust because of the low angular velocities involved. In the abrupt rudder kick, the accelerating moment is given by the equation

$$N = q_t m_t (k\delta) S_t x_t \quad (19)$$

where

- q_t dynamic pressure at tail.
- m_t slope of tail lift curve, radian measure.
- S_t vertical tail area.
- x_t tail length
- $(k\delta)$ effective change in angle of attack of vertical tail due to a rudder deflection, radians.

An analysis of the measured moments of inertia for about 20 conventional single-engine airplanes using the following assumptions

$$I_{\text{wings about X}} \approx I_{\text{wings about Z}}$$

$$I_{\text{fuselage about Y}} \approx I_{\text{fuselage about Z}}$$

$$I_{\text{fuselage about X}} \approx I_{\text{wings about Y}}$$

reveals that, on the average, the moments of inertia of the wings about the Z axis are equal to the moment of inertia of the fuselage about the same axis. Thus with conventional airplanes, approximately one-half the moment given by equation (19) would reach the wing-fuselage fittings and the rest would be absorbed by the inertia of the fuselage.

During a side motion of the airplane or a side gust, the action of the wind on the fuselage and the vertical tail is such as to cause a yawing moment

$$N = q_t M_t \frac{v}{V} S_t x_t \quad (19')$$

where v is the side velocity and V is the forward velocity. Whether this moment is larger than the abrupt rudder kick depends upon the relative magnitudes of $(k\delta)$ and v/V and also upon the wing dihedral. If the wing has positive dihedral, a side velocity produces a yawing moment which is proportional to both the dihedral angle Γ and the wing angle of attack and which is in the same direction as that produced by the vertical tail. (See reference 2, fig. 21.) For conventional arrangements, the effect of the wing dihedral is small relative to that of the fuselage and the tail. In a sideslip or a gust it should be possible, if the ratio of wing moment to wing inertia equaled the ratio of tail moment to fuselage inertia, for no twisting force to be transmitted to the wing-fuselage fittings.

In the landing condition, the magnitude of the horizontal component of force at the wheel is generally expressed as a fraction of the vertical landing-gear load factor so that the accompanying yawing moment would be, assuming this fraction as $1/4$,

$$N = \frac{n_v}{4} W y_t \quad (20)$$

Approximately one-half this moment would reach the wing-fuselage fittings.

Of the various possibilities considered in the yawing condition, the yawing moment caused by an abrupt rudder kick, or whatever yawing moment would result when the design load is applied to the vertical tail, appears to produce the most severe twist at the wing-fuselage fittings.

Langley Memorial Aeronautical Laboratory,
National Advisory Committee for Aeronautics,
Langley Field, Va., March 11, 1940.

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2. Pearson, Henry A., and Jones, Robert T.: Theoretical Stability and Control Characteristics of Wings with Various Amounts of Taper and Twist. T.R. No. 635, N.A.C.A., 1938.
3. Pearson, Henry A.: Approximate Span-Load Distribution for Preliminary Design. Jour. Aero. Sci., vol. 4, no. 9, July 1937, pp. 372-374.
4. Abbott, Ira H., and Greenberg, Harry: Tests in the Variable-Density Wind Tunnel of the N.A.C.A. 23012 Airfoil with Plain and Split Flaps. T.R. No. 661, N.A.C.A., 1939.

TABLE I

Shear Distribution Due to Type A Air Load $\frac{Q_{a_z}}{nW}$

λ $y/\frac{b}{2}$	4 : 1	2 : 1	4 : 3	1 : 1
0	0.5000	0.5000	0.5000	0.5000
.1	.4260	.4540	.4400	.4450
.2	.3590	.3710	.3830	.3890
.3	.2930	.3100	.3230	.3320
.4	.2330	.2530	.2670	.2770
.5	.1770	.1980	.2130	.2220
.6	.1280	.1470	.1600	.1690
.7	.0840	.1000	.1100	.1170
.8	.0470	.0590	.0650	.0700
.9	.0170	.0230	.0250	.0260
.95	.0060	.0090	.0100	.0110
1.0	0	0	0	0

TABLE II. Shear Distribution Due to Aileron Air Load $\frac{Q_a \alpha}{I_X \frac{b}{b}}$

b_a/b	1.00			0.75			0.50		0.25
λ	4:1	2:1	1:1	4:1	2:1	1:1	4:1	2:1	4:1 to 1:1
$y/\frac{b}{2}$									
0	2.128	2.050	1.983	1.790	1.736	1.700	1.465	1.440	1.238
.1	1.990	1.921	1.865	1.772	1.725	1.692	1.458	1.433	1.235
.2	1.742	1.705	1.675	1.714	1.677	1.647	1.440	1.417	1.227
.3	1.480	1.470	1.460	1.564	1.542	1.526	1.413	1.390	1.213
.4	1.200	1.223	1.235	1.300	1.303	1.305	1.358	1.340	1.188
.5	.942	.975	1.000	1.020	1.033	1.055	1.242	1.253	1.153
.6	.688	.725	.760	.743	.776	.805	.978	1.015	1.104
.7	.470	.495	.523	.500	.523	.553	.675	.715	1.010
.8	.262	.282	.313	.284	.300	.326	.385	.410	.728
.9	.083	.101	.117	.100	.102	.118	.130	.148	.280
.95	.027	.040	.043	.033	.040	.050	.045	.052	.100
1.0	0	0	0	0	0	0	0	0	0

TABLE III

Shear Distribution Due to Wing Weight

And Normal Inertia $\frac{Q_{WZ}}{fnW}$

λ $y/\frac{b}{2}$	4 : 1	2 : 1	4 : 3	1 : 1 ^a
0	0.5000	0.5000	0.5000	0.500
.1	.4057	.4185	.4331	.442
.2	.3229	.3432	.3704	.386
.3	.2510	.2795	.3117	.333
.4	.1894	.2212	.2569	.280
.5	.1375	.1697	.2057	.230
.6	.0947	.1246	.1581	.181
.7	.0602	.0855	.1139	.133
.8	.0335	.0520	.0729	.087
.9	.0137	.0236	.0350	.043
.95	.0056	.0106	.0156	.021
1.0	0	0	0	0

^aEstimated

TABLE IV

Shear Distribution Due to Angular Inertia $\frac{Q_W \alpha}{\frac{fW}{g} b\alpha}$

λ $y/\frac{b}{2}$	4 : 1	2 : 1	4 : 3	1 : 1
0	0.0885	0.0982	0.1099	0.1186
.1	.0861	.0962	.1083	.1173
.2	.0799	.0908	.1036	.1133
.3	.0710	.0826	.0964	.1067
.4	.0602	.0724	.0869	.0976
.5	.0486	.0609	.0754	.0861
.6	.0367	.0485	.0623	.0726
.7	.0255	.0358	.0480	.0570
.8	.0155	.0232	.0327	.0396
.9	.0072	.0112	.0166	.0206
.95	.0033	.0055	.0083	.0105
1.0	0	0	0	0

TABLE V

Moment Distribution Due to Type A Air Load $\frac{M_{az}}{nWb}$

λ $y/b/2$	4 : 1	2 : 1	4 : 3	1 : 1
0	0.1002	0.1068	0.1115	0.1145
.1	.0772	.0835	.0880	.0909
.2	.0574	.0634	.0674	.0701
.3	.0412	.0464	.0499	.0520
.4	.0280	.0324	.0351	.0368
.5	.0178	.0211	.0231	.0244
.6	.0103	.0126	.0138	.0148
.7	.0051	.0065	.0071	.0077
.8	.0019	.0025	.0028	.0030
.9	.0003	.0004	.0005	.0006
.95	.0001	.0001	.0001	.0002
1.0	0	0	0	0

TABLE VI. Moment Distribution Due to Aileron Air Load $\frac{M_{a\alpha}}{I_X \alpha}$

b_a/b	1.00			0.75			0.50		0.25
λ	4:1	2:1	1:1	4:1	2:1	1:1	4:1	1:1	4:1 to 1:1
$y/\frac{b}{2}$									
0	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000
.1	.3958	.4000	.4029	.4100	.4126	.4145	.4250	.4285	.4375
.2	.3016	.3082	.3139	.3219	.3260	.3296	.3515	.3548	.3734
.3	.2200	.2280	.2350	.2385	.2455	.2488	.2800	.2850	.3112
.4	.1524	.1600	.1672	.1660	.1715	.1767	.2080	.2136	.2496
.5	.1000	.1048	.1090	.1080	.1115	.1175	.1413	.1473	.1915
.6	.0577	.0630	.0665	.0628	.0659	.0702	.0844	.0895	.1320
.7	.0265	.0314	.0345	.0325	.0330	.0350	.0430	.0450	.0780
.8	.0101	.0130	.0133	.0117	.0126	.0136	.0155	.0171	.0320
.9	.0016	.0024	.0026	.0020	.0022	.0026	.0029	.0033	.0062
.95	.0005	.0005	.0005	.0005	.0005	.0005	.0005	.0005	.0010
1.0	0	0	0	0	0	0	0	0	0

TABLE VII

Moment Distribution Due to
Wing Weight and Normal Inertia $\frac{M_{wz}}{fnWb}$

λ y/b 2	4 : 1	2 : 1	4 : 3	1 : 1
0	0.0876	0.0965	0.1102	0.1185
.1	.0650	.0740	.0870	.0948
.2	.0468	.0553	.0669	.0740
.3	.0325	.0400	.0499	.0561
.4	.0215	.0277	.0356	.0408
.5	.0134	.0181	.0241	.0280
.6	.0076	.0109	.0150	.0178
.7	.0038	.0058	.0082	.0099
.8	.0015	.0024	.0036	.0048
.9	.0003	.0006	.0009	.0011
.95	.0001	.0002	.0002	.0003
1.0	0	0	0	0

TABLE VIII

Moment Distribution Due to Angular Inertia $\frac{M_W \alpha}{\frac{fW}{g} ab^2}$

λ $y/\frac{b}{2}$	4 : 1	2 : 1	4 : 3	1 : 1
0	0.02370	0.02861	0.03440	0.03866
.1	.01932	.02373	.02892	.03275
.2	.01515	.01902	.02361	.02698
.3	.01138	.01467	.01859	.02147
.4	.00812	.01079	.01400	.01635
.5	.00540	.00745	.00993	.01174
.6	.00328	.00471	.00647	.00775
.7	.00173	.00261	.00370	.00449
.8	.00071	.00114	.00167	.00206
.9	.00016	.00027	.00043	.00054
.95	.00003	.00006	.00011	.00014
1.0	0	0	0	0

TABLE IX

Shear Distribution Due to Trapezoidal Gust $\frac{Q_{ag}}{I_x \frac{\alpha}{b}}$

λ $y/\frac{b}{2}$	4 : 1	2 : 1	4 : 3	1 : 1
0	1.710	1.678	1.655	1.625
.1	1.681	1.653	1.633	1.605
.2	1.596	1.579	1.565	1.544
.3	1.461	1.459	1.456	1.443
.4	1.286	1.300	1.308	1.304
.5	1.080	1.108	1.125	1.128
.6	.852	.888	.909	.917
.7	.612	.648	.670	.678
.8	.371	.399	.416	.421
.9	.145	.161	.169	.171
.95	.052	.060	.064	.065
1.0	0	0	0	0

TABLE X

Moment Distribution Due to Trapezoidal Gust $\frac{M_{ag}}{I_x \alpha}$

λ $y/\frac{b}{2}$	4 : 1	2 : 1	4 : 3	1 : 1
0	0.5000	0.5000	0.5000	0.5000
.1	.4160	.4170	.4183	.4196
.2	.3316	.3360	.3382	.3405
.3	.2543	.2605	.2631	.2658
.4	.1856	.1919	.1945	.1972
.5	.1261	.1319	.1341	.1364
.6	.0774	.0821	.0837	.0854
.7	.0406	.0437	.0447	.0457
.8	.0158	.0173	.0178	.0182
.9	.0031	.0034	.0035	.0037
.95	.0006	.0007	.0007	.0007
1.0	0	0	0	0

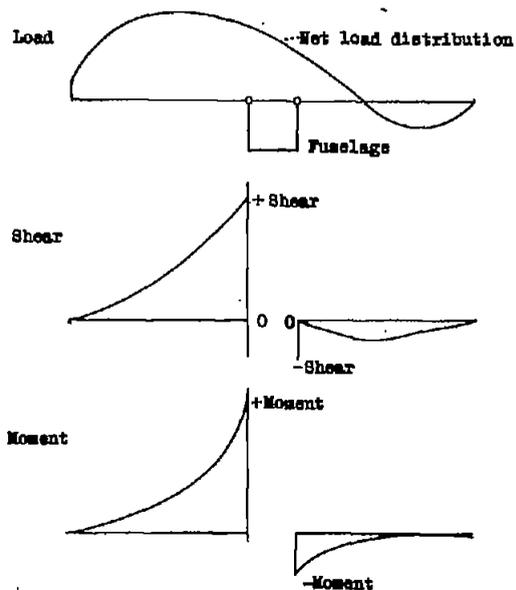
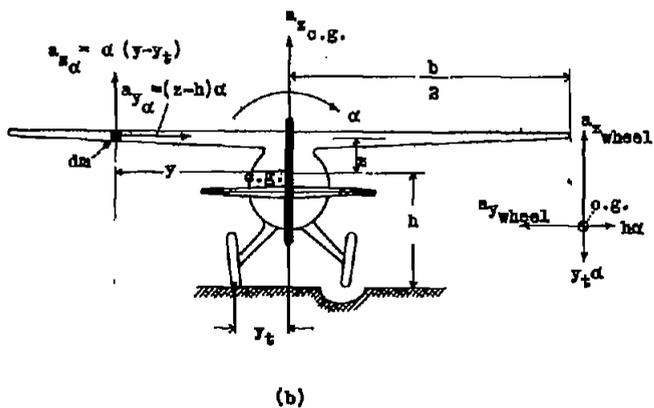
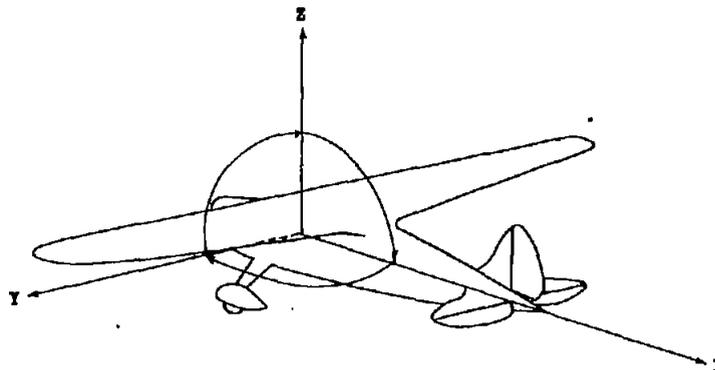
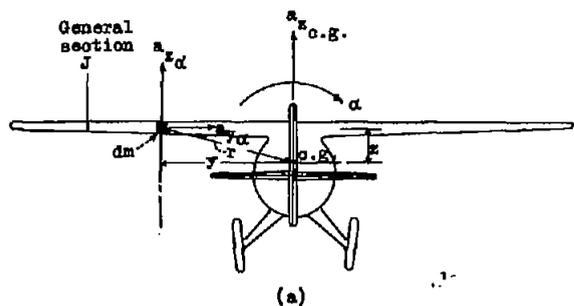
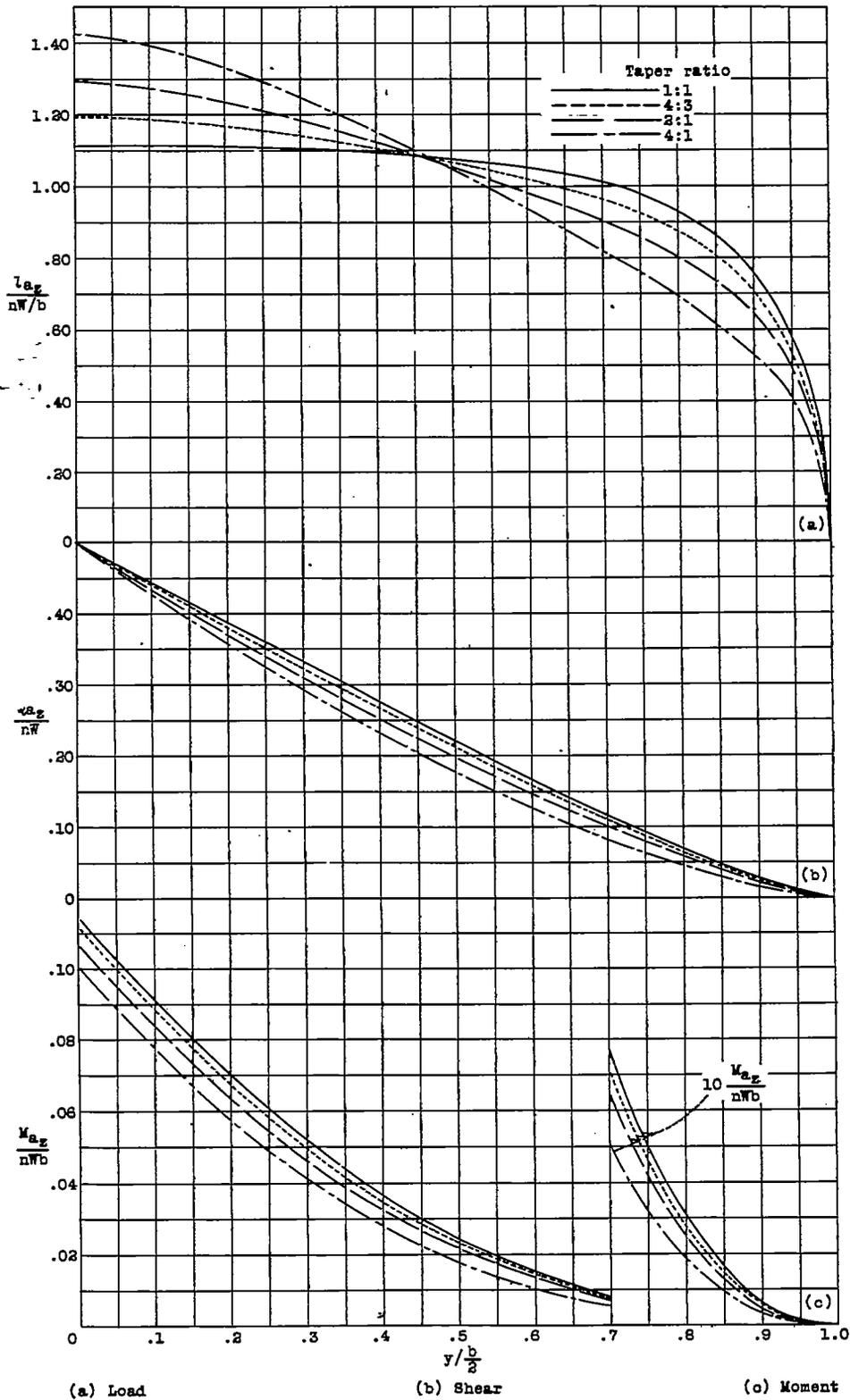


Figure 2.- Accelerations on airplanes.

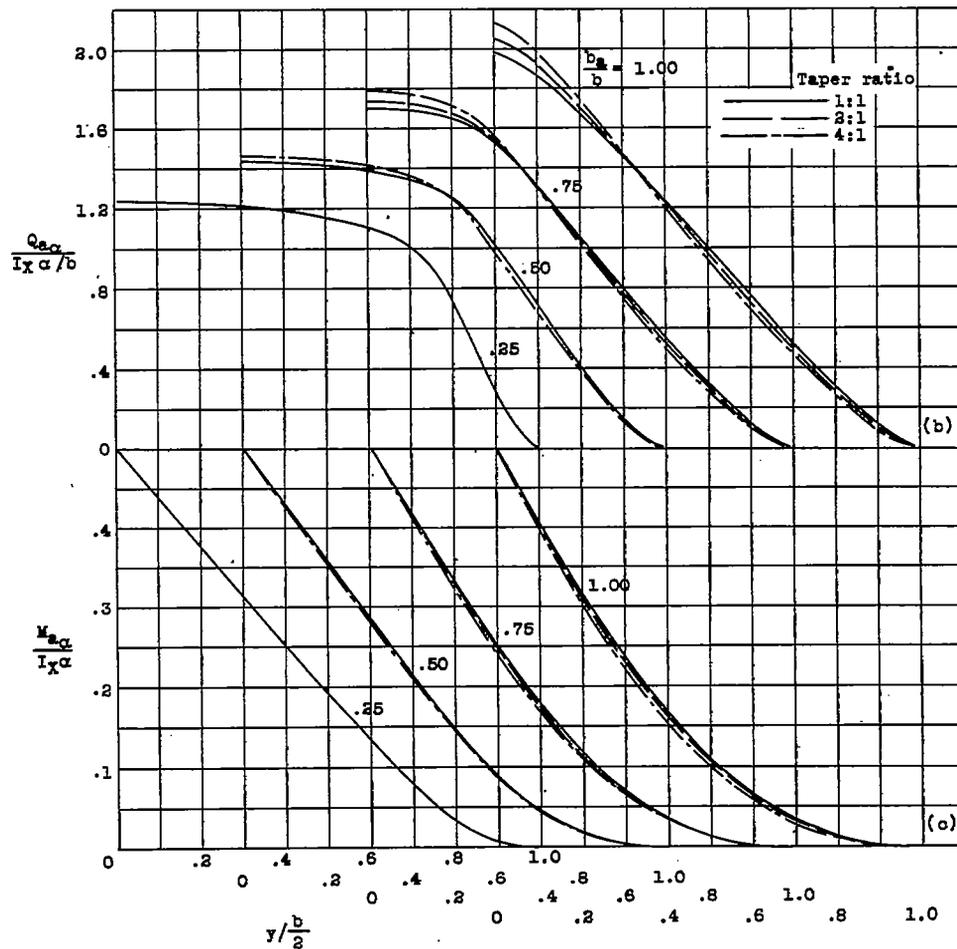
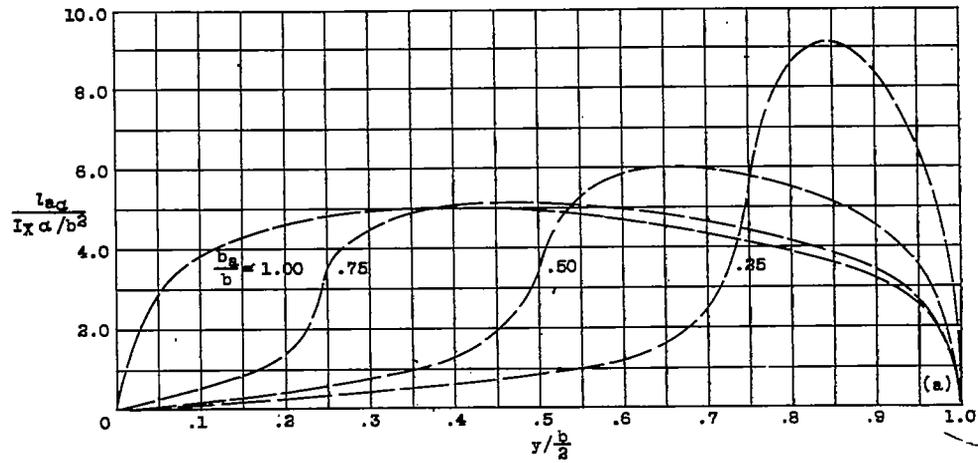
Figure 1.- Sign conventions.

$$c_a = \frac{c_a A}{b C_u}$$



(a) Load (b) Shear (c) Moment

Figure 3.- Load, shear, and moment distribution for type A air load.



(a) Load $\lambda = 2:1$

(b) Shear

(c) Moment

Figure 4.- Load, shear, and moment distribution due to aileron air load.

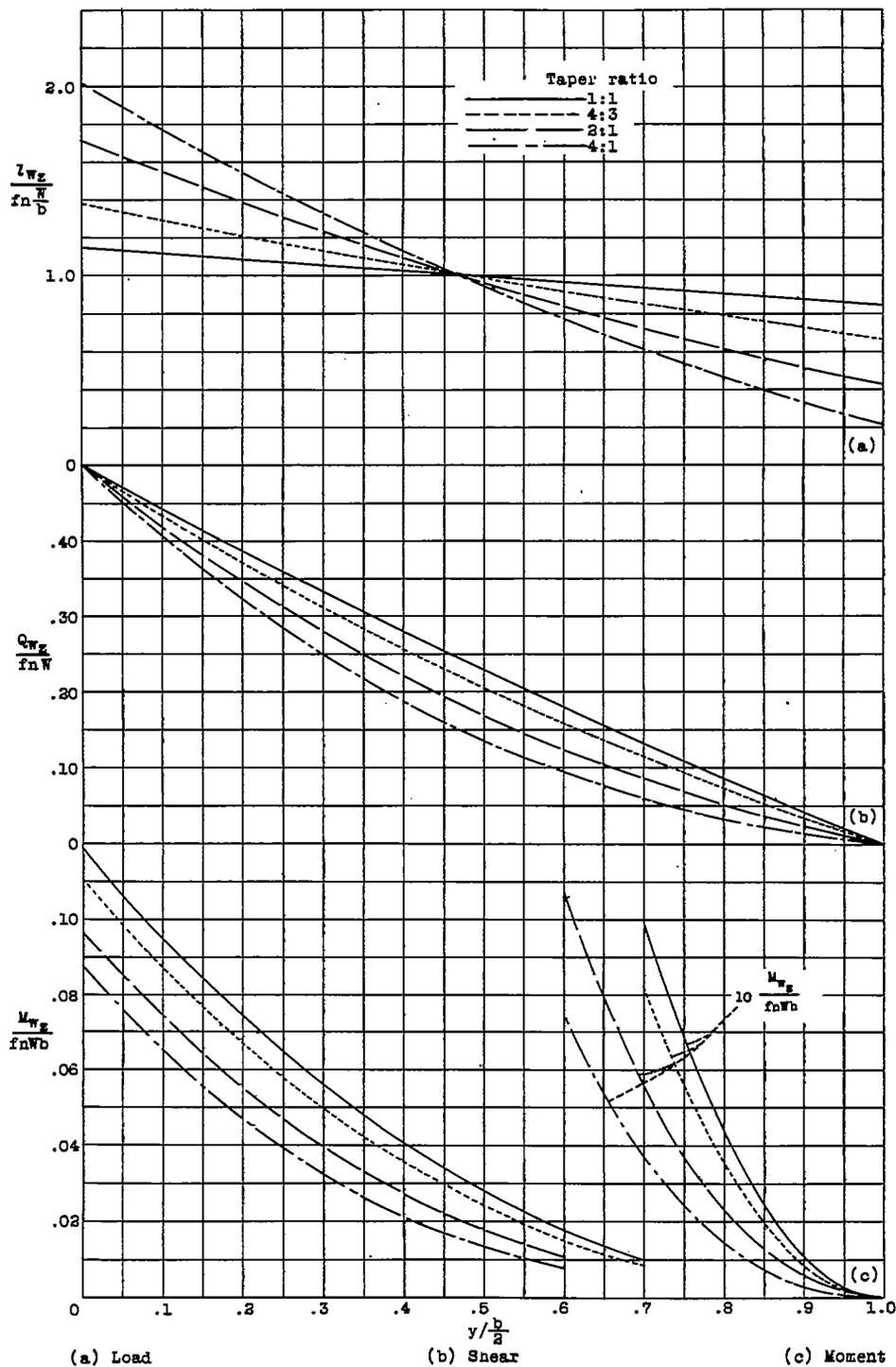


Figure 5.- Load, shear, and moment distribution due to wing weight and normal inertia load.

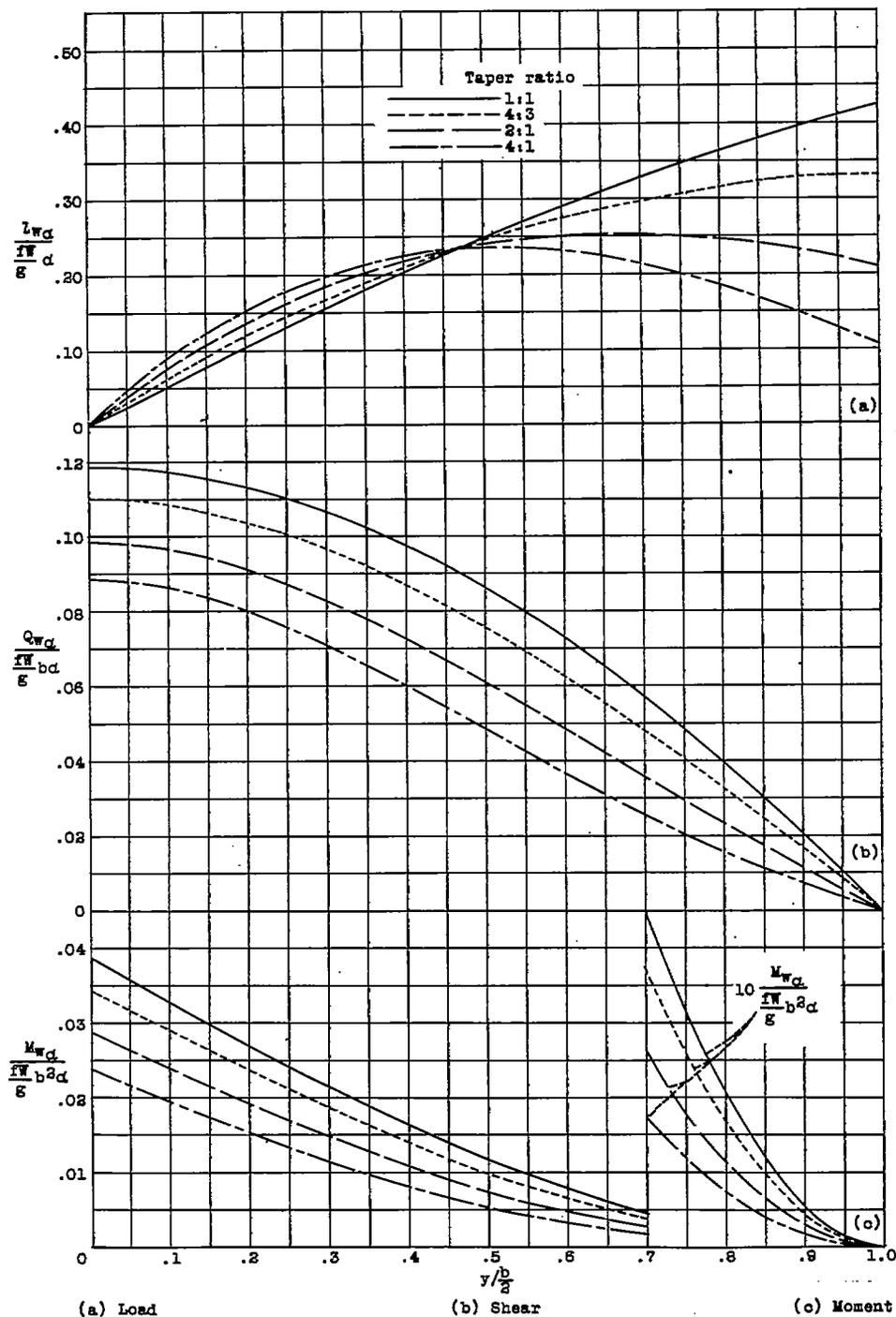


Figure 6.- Load, shear, and moment distribution due to angular inertia load.

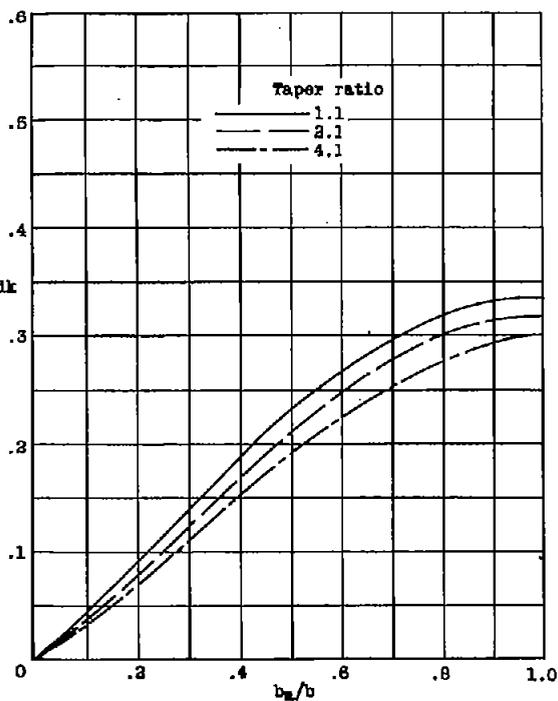


Figure 7.- Angular acceleration factor for ailerons.

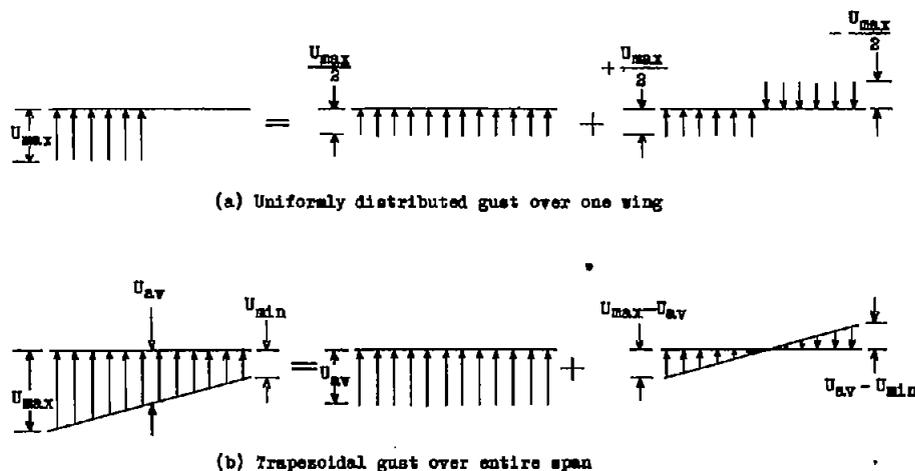


Figure 8.- Elementary gust types.

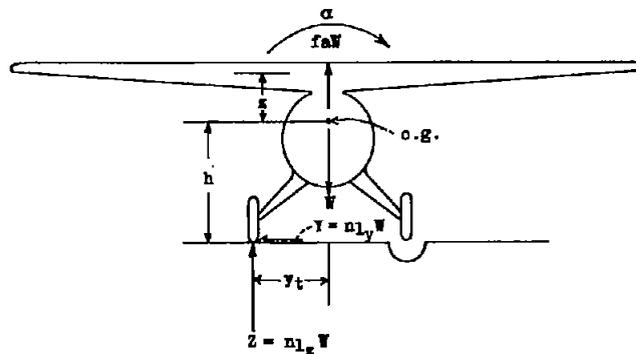


Figure 10.- Forces acting in a one-wheel landing.

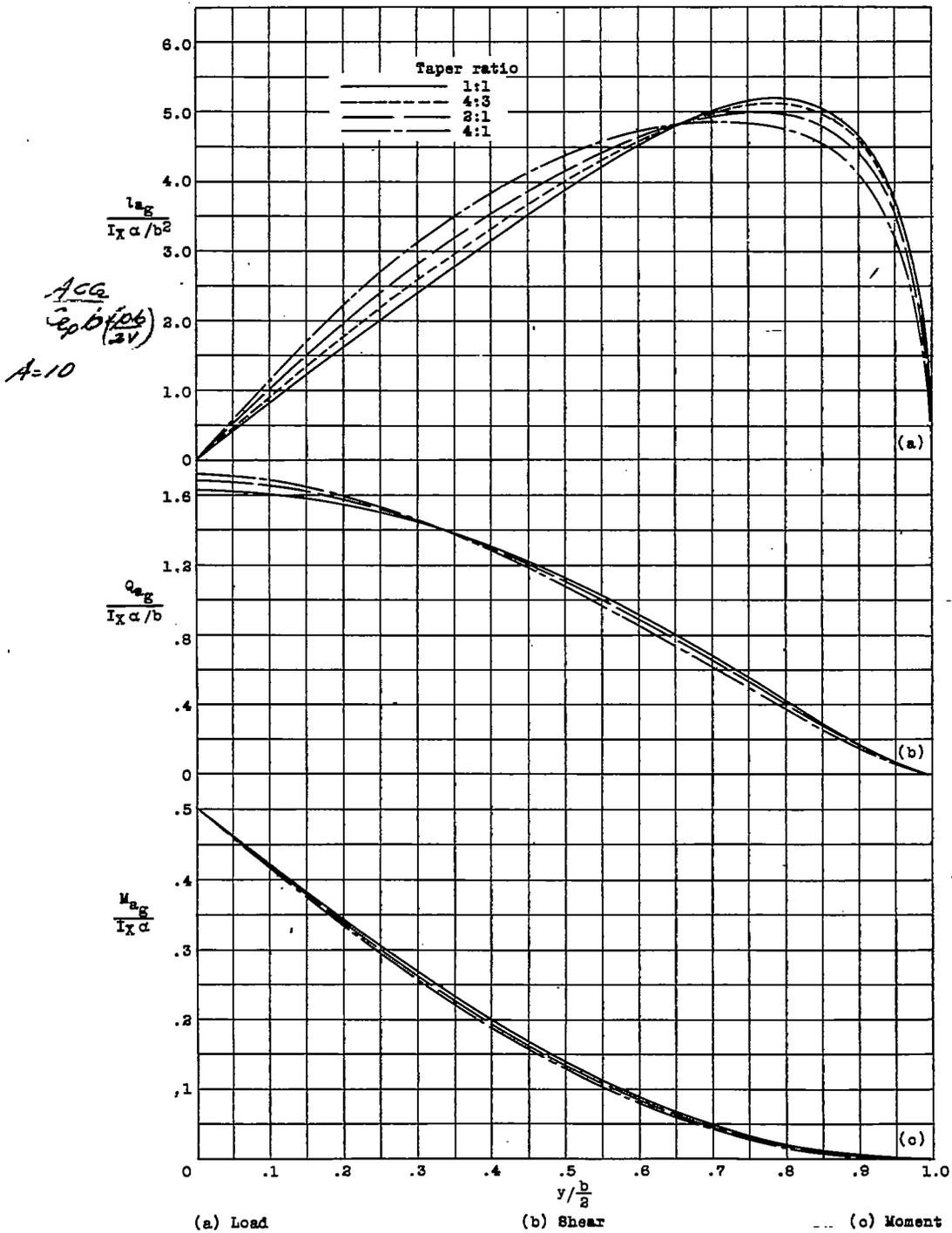


Figure 9.- Load, shear, and moment distribution due to a trapezoidal gust.

- Type A air load
- Normal weight and inertia load
- Net symmetrical load
- Aileron air load
- Angular inertia load
- Net unsymmetrical load

Components dependent on n

Components dependent on α

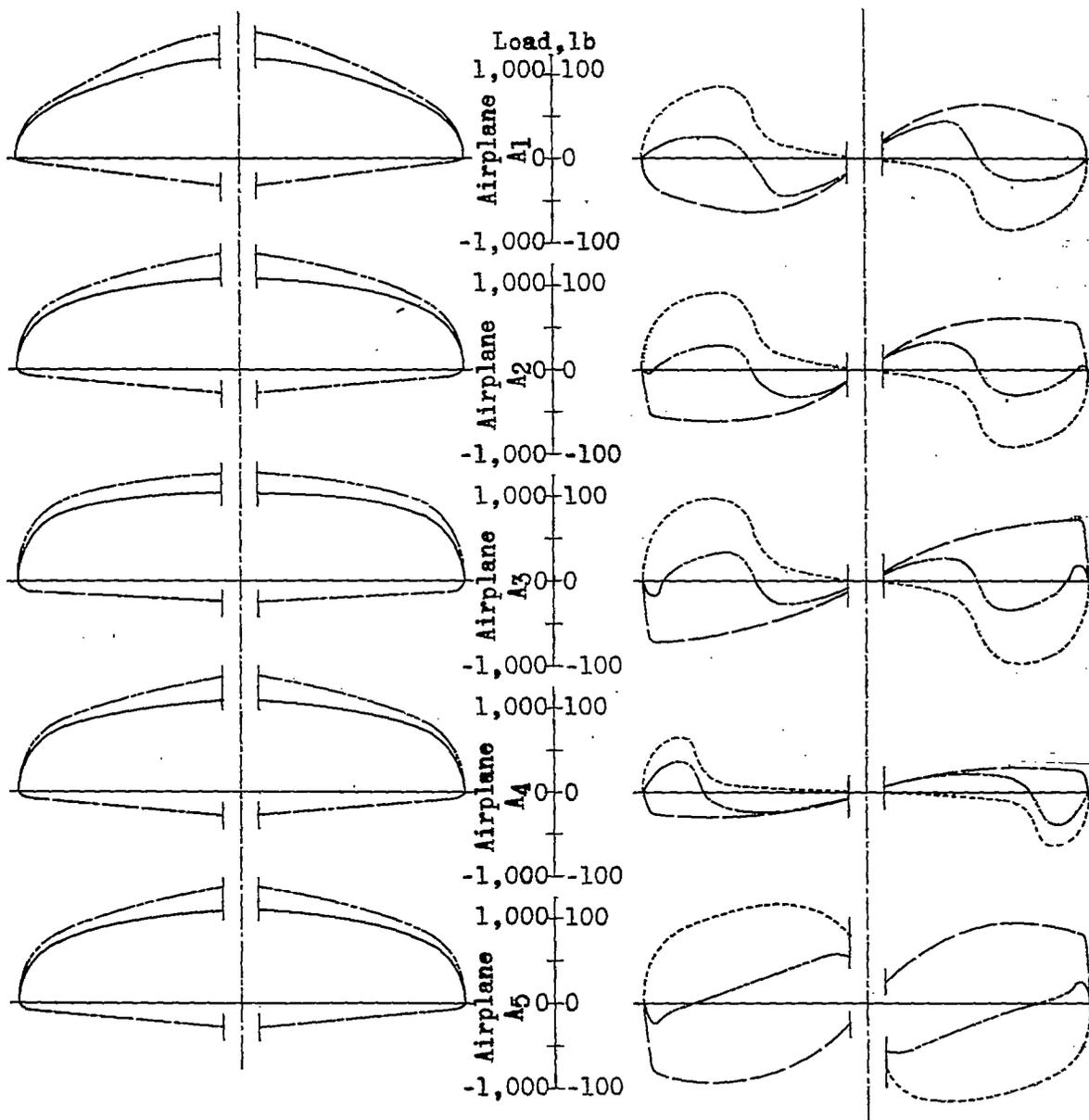


Figure 11.- Load components for airplanes A₁ to A₅ in flight.

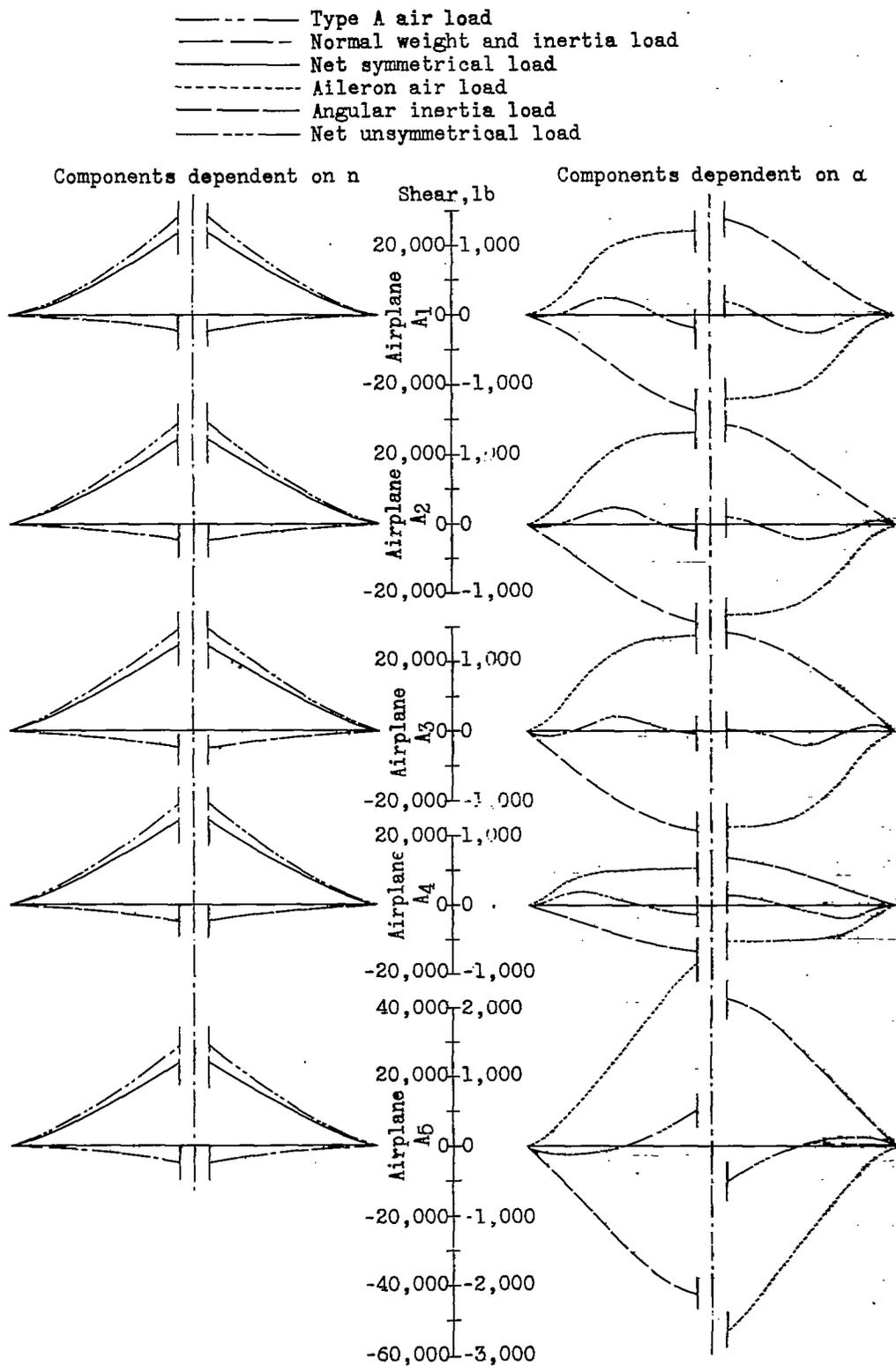


Figure 12.- Shear components for airplanes A₁ to A₅ in flight.

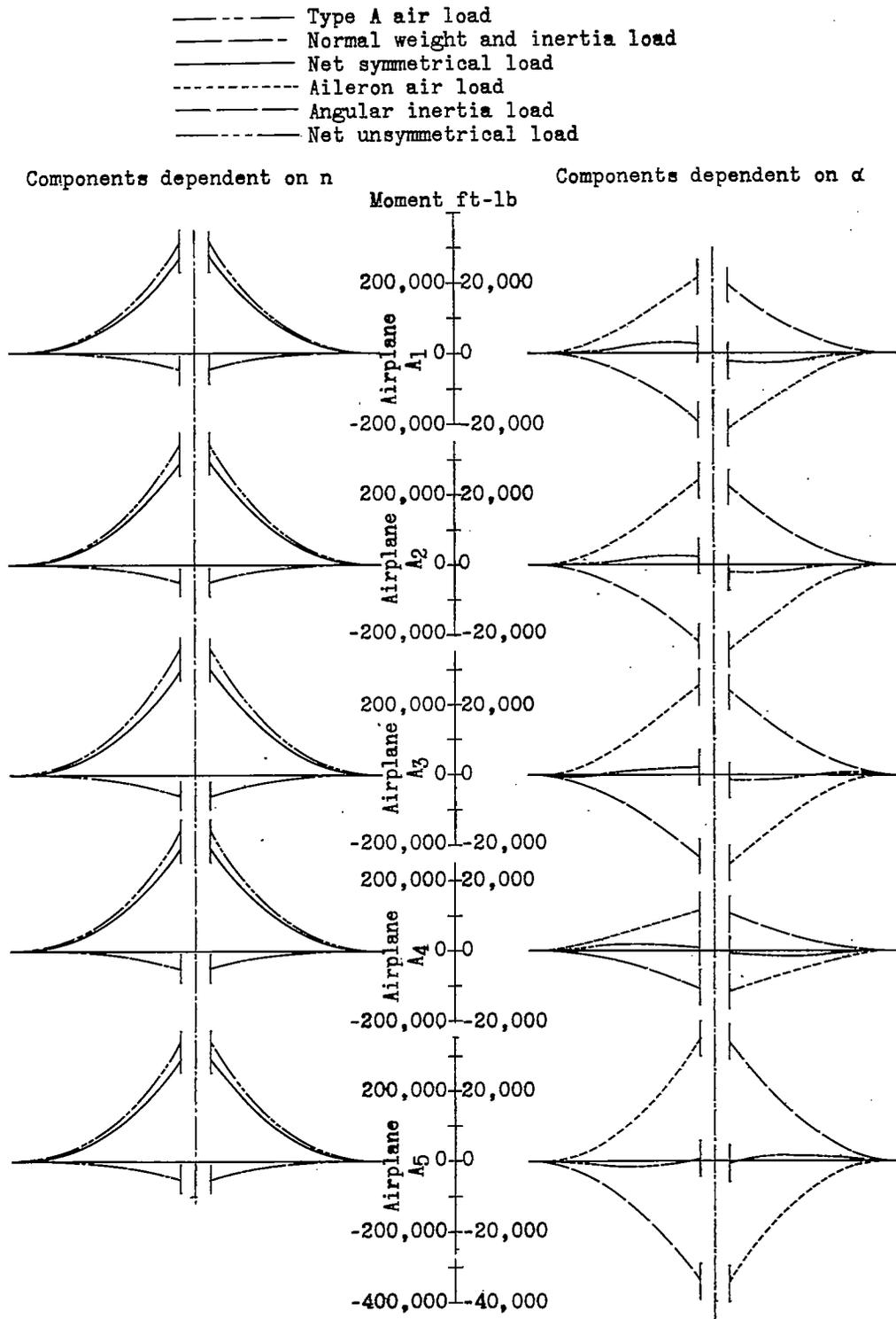


Figure 13.- Moment components for airplanes A₁ to A₅ in flight.

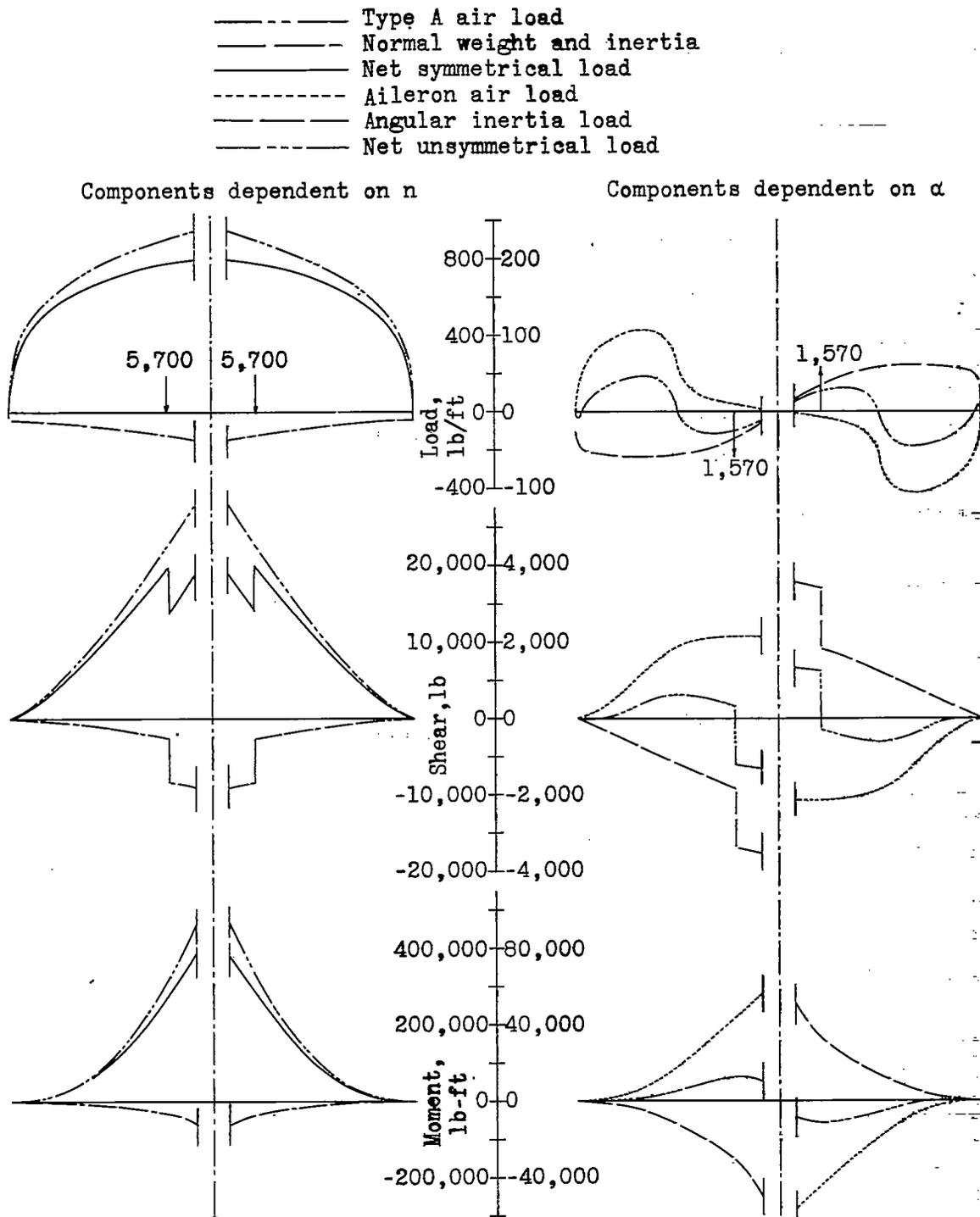


Figure 14.- Component load, shear, and moment distributions for airplane B.

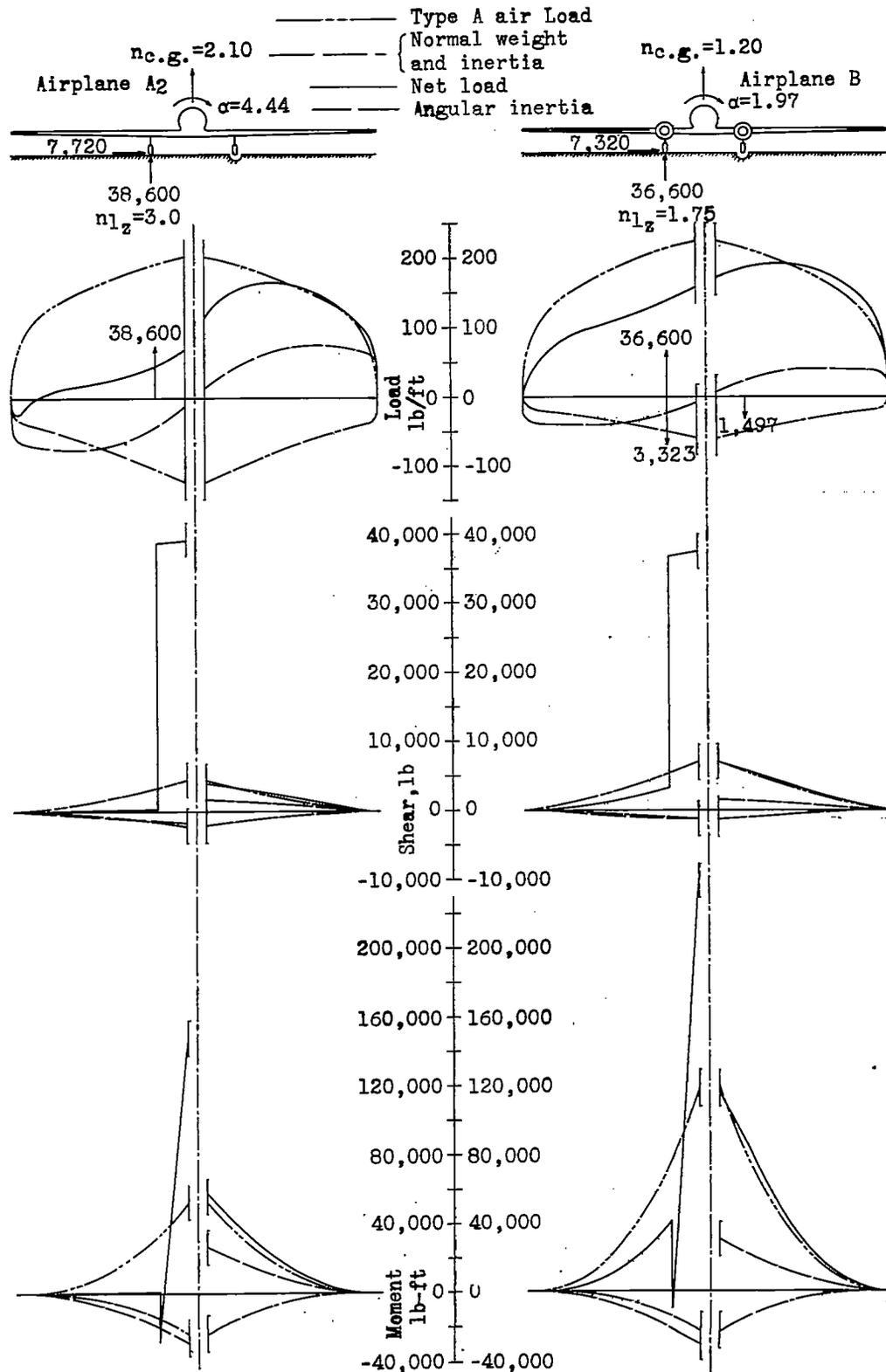


Figure 15.- Component load, shears, and moments in one-wheel landing.